Code No. 16102/Core

FACULTY OF SCIENCE

M.Sc. I Semester Examination, September 2021
Subject: Mathematics/Applied Mathematics/MCS
Paper – I: Algebra

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 If G is a group and H<G of finite index n then prove that there is a homomorphism $\phi : G \to S_n$ such that Ker$\phi = \bigcap_{x \in G} xHx^{-1}$.

2 Prove that a group of order p^n is nilpotent where p is a prime.

3 If G is a group of order mn where (m,n)=1 then prove that $G \simeq$HxK where H,K are subgroups of order m and n respectively.

4 If a group of order p^n contains exactly one sub group each of orders $p, p^2, \ldots p^{n-1}$ then prove that it is cyclic.

5 Prove that the only homomorphisms from the ring of integers Z to Z are the identity and zero mappings.

6 Prove that a commutative ring R with unity in which each ideal is prime is a field.

7 Prove that every Euclidean domain is a PID.

8 Define regular element and multiplicative set in a ring R. Show that the set of all regular elements is a regular multiplicative set.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Prove that the set Aut(G) of all automorphisms of a group G is a group under composition of mappings and In(G)$\lhd$ Aut(G).

10 For a nilpotent group G, prove that every subgroup of G and every homomorphic image of G are nilpotent.

11 State and prove Cauchy's theorem for abelian groups.

12 State and prove sylow second and third theorems.

13 (i) If f:R→S is a homomorphism of a ring R into a ring S then prove that Ker $f = \{0\}$ if and only if f is one-one.

(ii) If $A_1, A_2 \ldots A_n$ are right ideals in a ring R then prove that the following are equivalent:

(1) $A = \sum_{i=1}^{n} A_i$ is a direct sum

(2) If $0 = \sum_{i=1}^{n} a_i,\ a_i \in A_i$ then $a_i = 0$ for $i = 1,2 \ldots n$

(3) $A_i \bigcap \sum_{\substack{j=1 \\ j \neq i}}^{n} A_j = \{0\}$ for $i = 1,2 \ldots n$.

14 In a commutative ring R prove that an ideal P is prime if and only if $ab \in P$, $a, b \in R$ implies $a \in P$ or $b \in P$.

15 Prove that every PID is a UFD.

16 If R is a UFD then prove that the polynomial ring $R[x]$ over R is also a UFD.

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Code No. 16103/Core

FACULTY OF SCIENCE

M.Sc. I Semester Examination, September 2021

Subject: Maths/Applied Maths/Maths with Computer Science
Paper – II: Mathematical Analysis

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Define convex set. Show that open balls are convex.
2 Prove that compact subsets of metric spaces are closed.
3 Prove that composition of two continuous functions is continuous.
4 Prove that the number of discontinuities of a monotonic function is atmost countable.

5 Prove that $\underline{\int_a^b} f d\alpha \le \overline{\int_a^b} f d\alpha$.

6 If $f(x) = x^2$ and $\alpha(x) = x^3$ then evaluate $\int_0^1 f d\alpha$.

7 Discuss with an example that a convergent series of continuous functions may have a discontinuous sum.

8 If $f_n \in R(\alpha)$ on $[a,b]$ and if $f(x) = \sum_{n=1}^{\infty} f_n(x),\ x \in [a,b]$. If the series $\sum_{n=1}^{\infty} f_n$ converges conformely to f on $[a,b]$, then prove that $\int_a^b f d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n d\alpha$.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Suppose $Y \subseteq X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X .

10 If P is a non empty prefect set in IR^k then prove that P is uncountable.

11 (i) Suppose f is a bijective continuous mapping defined on a compact metric space X into Y .
Then prove that the inverse f^{-1} defined by $f^{-1}(f(x)) = x,\ \forall\ x \in X$ is also continuous.
(ii) Discuss with an example that if compactness is relaxed in the above statement, then f^{-1} need not be continuous.

12 Suppose (X, d_1) is a compact metric space and (Y, d_2) is any metric space. If $f : X \to Y$ continuous, then show that f if uniformly continuous.

..2..

13 Let f be a bounded function defined on $[a,b]$ which has a finite number of discontinuities on [a,b]. Let $\{y_1, y_2, y_3 \ldots y_r\}$ be the set of discontinuities of f on $[a,b]$. If α is continuous at y_i for all i =1,2... n, then $f \in R(\alpha)$ on $[a,b]$.

14 Suppose f is a bounded real valued function defined on $[a,b]$. If α is a monotonically increasing and differentiable on $[a,b]$ such that $\alpha' \in R$ on $[a,b]$. Then prove that $f \in R(\alpha)$ on $[a,b] \Leftrightarrow f\alpha' \in R$ on $[a,b]$. Also $\int_a^b f d\alpha = \int_a^b f\alpha' dx$.

15 Let $\{f_n\}$ be a sequence of continuous functions defined on a compact set K such that

(i) $\{f_n\}$ converges point wise to a continuous function f on K

(ii) $f_n(x) \geq f_{n+1}(x), \forall\ n, \forall x \in K$, then $f_n \to f$ uniformly on K.

16 State and prove Weierstrass approximation theorem.

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14 Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$.

15 Prove that $\exp\left\{\frac{x}{2}\left(z - \frac{1}{z}\right)\right\} = \sum_{n=-\infty}^{\infty} z^n J_n(x)$.

16 Prove that $\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\,dx = 0$ if $m \neq n$

$= \sqrt{\pi}\, 2^n n!$ if $m = n$.

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Code No. 16106/Core

FACULTY OF SCIENCE

M.Sc. I Semester Examination, September 2021
Subject: Mathematics
Paper – IV: Elementry Number Theory

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 If d / n and d / m, prove that $d /(an + bm)$, where a, b are integers and also prove that $|d| = |n|$.

2 Find the gcd of 1256 and 2158 by factorizing them into distinct prime powers.

3 If $n \geq 1$, prove that $\sum_{d/n} \mu(d) = \left[\frac{1}{n}\right] = \begin{cases} 1 & if\ n = 1 \\ 0 & if\ n > 1 \end{cases}$.

4 Prove that Dirichlet multiplication of arithmetical functions is commutative.

5 If $a \equiv b \pmod{m}$ and $\alpha \equiv \beta \pmod{m}$, then prove that
(i) $a\alpha \equiv b\beta \pmod{m}$ (ii) $ax + \alpha y \equiv bx + \beta y \pmod{m}$ for all integers x and y.

6 State and prove Euler-Fermat theorem.

7 Find quadratic non-residues modulo 17.

8 Evaluate (18/31).

PART – B

Note: Answer any Three questions. **(3 x 15 = 45 Marks)**

9 (i) Given integers "a" and "b" with b >0, prove that there exists a unique pair of integers q and r such that $a = bq + r$, where $0 \leq r < b$. Moreover $r = 0$ if b / a.

(ii) If a = 5182, b = 105, find integers q and r such that $a = bq + r$ with $0 \leq r < b$

10 (i) Prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges, where p_n is n^{th} prime.

(ii) Prove that there are infinitely many prime numbers.

11 If both g and $f * g$ are multiplicative then prove that f is also multiplicative.

12 State and prove Mobius inversion formula.

13 (i) Assume gcd (a,m)=d and suppose that d/b. Then prove that the linear congruence $ax \equiv b \pmod{m}$ has exactly d solutions modulo m, given by $t, t + \frac{m}{d}, t + 2\frac{m}{d}, \ldots, t + \frac{(d-1)m}{d}$, where t is the solution, unique modulo $\frac{m}{d}$, of the linear congruence $\frac{a}{d}x \equiv \frac{b}{d}\left(\mod \frac{m}{d}\right)$.

(ii) Solve the congruence $21x \equiv 14 \pmod{35}$.

.2..

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14 (i) State and prove Chinese remainder theorem.
(ii) State and prove Wolstenholme's theorem.

15 State and prove Gauss lemma.

16 For every odd prime p, prove that $(2/p) = (-1)^{\left(\frac{p^2-1}{8}\right)} = \begin{cases} 1, & if\ p \equiv \pm 1 \pmod 8 \\ -1, & if\ p \equiv \pm 3 \pmod 8 \end{cases}$ and hence find (2/173).

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Code No. 16109/Core

FACULTY OF SCIENCE

M.Sc. I Semester Examination, September 2021

Subject: Mathematics

Paper – V: Discrete Mathematics

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Express the following arguments symbolically using quantifiers.
"All humming birds are richly coloured"
"No large birds live on honey"
"Birds that do not live on honey are dull in color"
"Humming birds are small".

2 Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

3 Using Euclidean algorithm express the gcd of 414 and 662 as linear combination of 414 and 662.

4 Using induction prove that $n < 2^n$, n being positive integer.

5 How many ways are there to place 10 indistinquishable balls into eight distinguishable bins?

6 Solve the recurrence relation for $a_n^2 - 2a_{n-1}^2 = 1$ for $n \geq 1$, $a_o = 1$.

7 Write a short note on tree traversal.

8 Show that a complete graph k_n is planar if and only if $n \leq 4$.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Construct truth table for $[(p \vee q) \wedge \sim r] \leftrightarrow (q \rightarrow r)$

10 Show that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is tautology.

11 Find all solutions to the system of congruences
$x \equiv 7 \pmod{9}$, $x \equiv 4 \pmod{12}$, $x \equiv 16 \pmod{21}$.

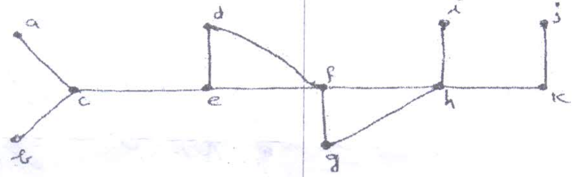
12 State and prove Chinese Remainder theorem.

13 State and prove principle of inclusion and exclusion on n sets $A_1, A_2, \ldots, A_n$.

14 Find the number of non-negative integral solutions to $x_1 + x_2 + x_3 = 11$ where x_1, x_2 and x_3 are non-negative integers with $x_1 \leq 3$, $x_2 \leq 4$ and $x_3 \leq 6$.

15 State and prove Euler's formula for planar graphs.

16 Using DFS algorithm find spanning tree for the following graph.



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